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Ising exponents in the two-dimensional site-diluted Ising model

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Abstract. We study the site-diluted Ising model in two dimensions with Monte Carlo simulations. Using finite-size scaling techniques we compute the critical exponents observing deviations from the pure Ising ones. The differences can be explained as the effects of logarithmic corrections, without requiring us to change the universality class.

Recently [1], it has been reported using Monte Carlo (MC) simulations, that the site-diluted Ising model in two dimensions seems to present a second-order transition line with concentration-dependent critical indices. Previously, other authors [2, 3] had claimed, using analytical methods, an Ising critical behaviour corrected with logarithms.

The field theoretical predictions for this model (based on renormalization-group and conformal-field theory) [4] assesses that disorder does not change the η or ν exponents since it only changes subleading terms. However, the specific heat, at the critical point, diverges as $\log(\log L)$ in the disordered system, while it does as $\log L$ in the pure case, L being the lattice size.

In this paper, we extend the methods developed in [5] for the four-dimensional site-diluted Ising model to the two-dimensional case.

We observe that, although an apparent variation of the indices seems to happen when varying the concentration, this can be explained as a transient effect. In fact, a pure Ising value for the indices plus logarithmic corrections fits our MC data very well. When preparing this paper, other authors have also reported similar conclusions in the bond-diluted Ising model using different techniques [6].

The numerical methods as well as the analytical computations based on the perturbative renormalization group are very similar in four and two dimensions, so we will report here just the minimal details required, concentrating on the description of the results: see [5] for further details of the method.

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Our procedure is based on a finite-size scaling (FSS) analysis. We perform MC simulations in the critical region for several values of the concentration p. For each p value, we generate hole configurations in a square lattice filling the sites randomly with probability p.

For each hole configuration (sample), we perform a MC simulation of the Ising model defined as the set of spins lying in the filled sites coupled through a nearest-neighbour interaction. In the smallest concentration we use the cluster Swendsen–Wang algorithm [7] to update the signs of the spins. For the remaining concentrations, we found the Wolff single-cluster version of this algorithm [8] to be more efficient. At each sample, we measure the energy, the magnetization and the Fourier transform of the Ising field at minimal momenta in about 100 independent spin configurations. We store the independent measures in order to compute derivatives with respect to the coupling and to extrapolate the results to close values of the coupling and the dilution. We have extrapolated in β in all cases but $p \simeq \frac{2}{3}$ where a p extrapolation performs better.

To reduce statistical errors, we have generated 10 000 hole samples for each (L, p) pair, L being the lattice size. We have simulated at concentrations: $p = 1, \frac{8}{9}, \frac{3}{5}, \frac{2}{3}$. Recall that the percolation threshold is at $p_c \simeq 0.59$ [9]. The FSS method that we use [10] is based upon the ratio of several observables: magnetization, susceptibility, correlation length, Binder cumulants, and their derivatives for two different lattice sizes L_1 and L_2. At the parameter values where the correlation lengths ratio matches L_2/L_1 we expect that, in the absence of logarithmic corrections

$$\frac{O(L_2, \beta, p)}{O(L_1, \beta, p)} = (L_2/L_1)^{x_O/\nu} + O(L_1^{-\omega}) \tag{1}$$

where x_O is the critical exponent for the observable O, e.g. γ for the susceptibility, ν for the correlation length, etc. ω is the universal corrections-to-scaling exponent. The β-derivatives of the previous observables go with the corresponding exponents plus $1/\nu$. In all cases we use pairs of lattices of sizes L and $2L$.

In tables 1 and 2 we report the results for the exponents ν and η respectively, using the relation (1) for the β-derivative of the correlation length in the former case and the susceptibility in the latter (after applying the scaling relation $\eta = 2 - \gamma/\nu$). Note that there is some statistical anticorrelation between sucessive even (odd) rows. We recall the critical exponents in the $\beta \to \infty$ limit (pure site percolation), $\nu = \frac{4}{3}$ and $\eta = \frac{5}{24}$ conjectured by Nienhuis [11] and in agreement with recent MC work [12].

Although the ν exponent seems to be non-constant as a function of the concentration, the values for η are very stable. This fact was also observed in the scaling of the Yang–Lee zeros at the critical point [13].

Table 1. The ν exponent for $(L, 2L)$ pairs at different dilutions.

L	$p = 1.0$	$p = 0.888\,89$	$p = 0.75$	$p = 0.6666$
24	1.009(4)			
32	1.009(5)	1.078(6)	1.142(9)	1.182(12)
48	0.995(4)	1.072(6)	1.114(9)	1.170(12)
64	1.001(4)	1.079(5)	1.112(9)	1.151(10)
96	1.010(5)	1.075(6)	1.111(9)	1.125(12)
128	0.993(4)	1.066(5)	1.098(8)	1.141(13)
192		1.065(6)	1.096(9)	1.140(13)
256	1.004(5)			

Table 2. The η exponent for $(L, 2L)$ pairs at different dilutions.

L	$p = 1.0$	$p = 0.888\,89$	$p = 0.75$	$p = 0.6666$
24	0.2465(7)			
32	0.2466(8)	0.2495(9)	0.2504(10)	0.2454(18)
48	0.2495(8)	0.2487(7)	0.2498(11)	0.2448(14)
64	0.2499(8)	0.2490(8)	0.2469(8)	0.2455(15)
96	0.2498(8)	0.2501(8)	0.2460(9)	0.2456(14)
128	0.2497(8)	0.2495(8)	0.2469(9)	0.2461(13)
192		0.2485(8)	0.2483(7)	0.2483(14)
256	0.2517(7)			

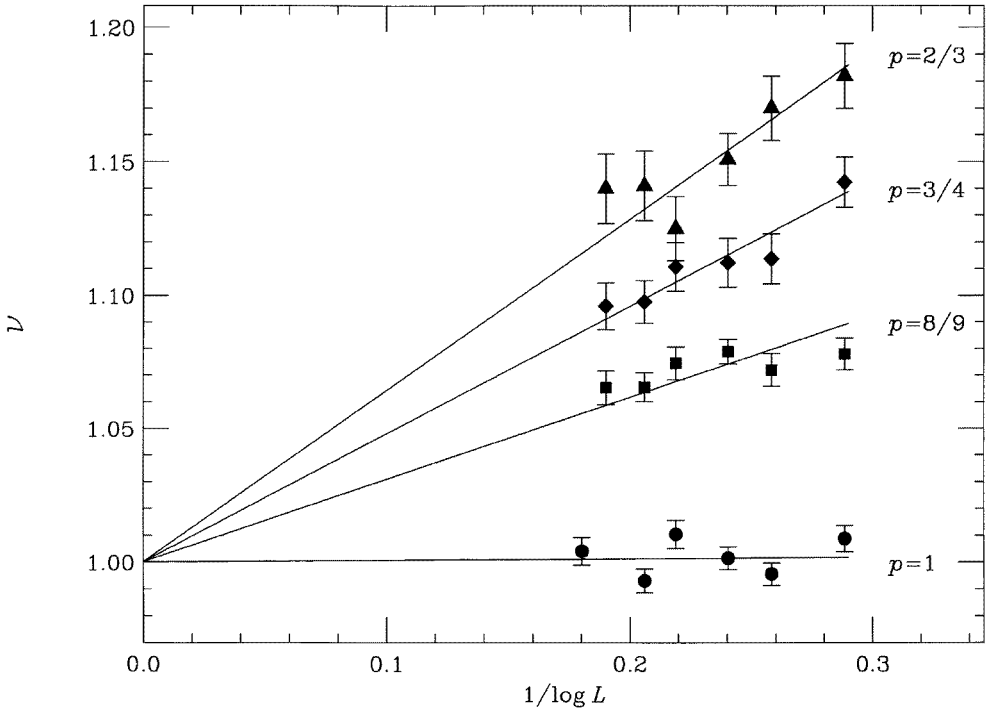

Figure 1. ν exponent as a function of $1/\log L$ for several values of p.

Table 3. Critical parameters for several dilutions. We also show the computed corrections to scaling exponent ω, the fit quality and the range of lattice sizes used (crossings from $L_1 - L_2^a$ to $L_1 - L_2^b$).

p_c	β_c	ω	χ^2/d.o.f.	$L_1 - (L_2^a - L_2^b)$
1	0.440 682(5)	1.5(8)	15.2/12	24 – (32 – 512)
0.888 89	0.537 81(2)	1.0(3)	2.6/8	48 – (64 – 384)
0.75	0.771 25(8)	0.9(5)	7.7/6	48 – (96 – 384)
0.666 61(3)	1.10	0.6(2)	9.6/8	32 – (64 – 384)

An interpretation of these results could be a continuous set of universality classes, as

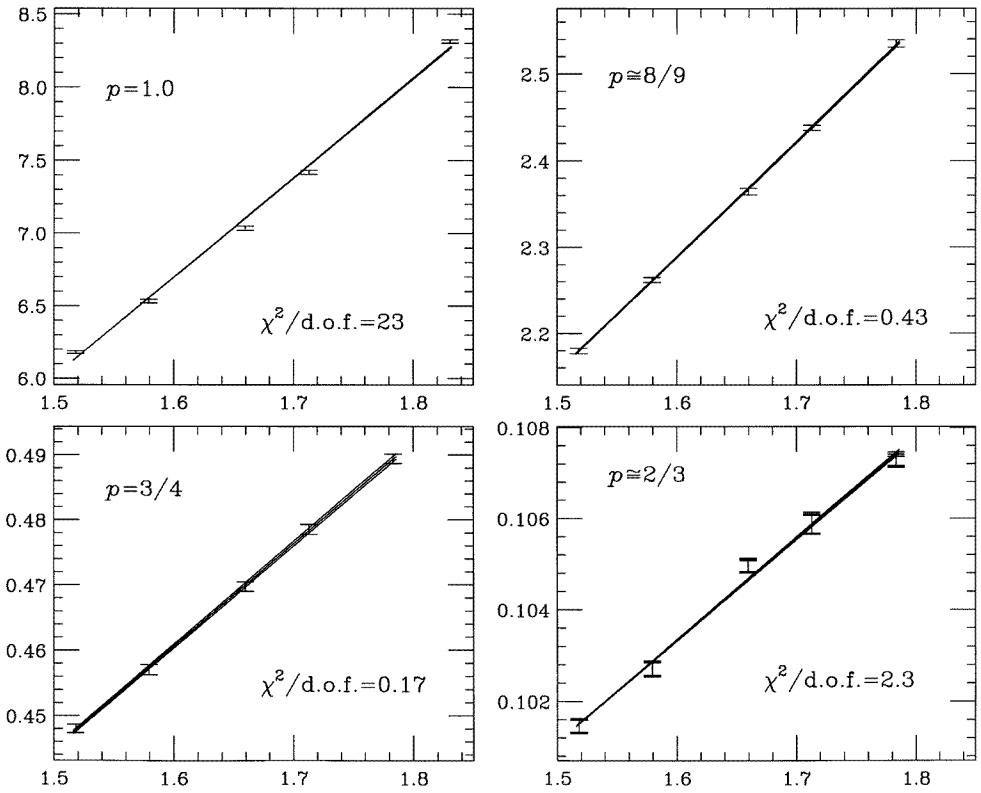

Figure 2. Specific heat as a function of the double logarithm of the lattice size. The lines correspond to linear fits for $L \geqslant 96$. The three lines for each case show the smallness of the effect of the error in the determination of the critical parameters.

assumed in [1]. However, a simpler scenario is a fixed value for ν (that of the Ising Model) plus logarithmic corrections.

Using the results from [2, 14] it is possible to show that the derivative of the correlation length with β reads at the critical point as

$$\partial_\beta \xi \propto -\frac{\xi^2}{\sqrt{1+C\log L}}\left(1+\frac{C}{2+2C\log L}\right) \tag{2}$$

where C depends on the dilution. Computing the ν exponent from (1) and (2) we obtain an apparent behaviour as

$$\nu^{\text{apparent}} = 1 + A/\log L + \cdots. \tag{3}$$

In figure 1 we plot the values of ν reported in table 2 as a function of $1/\log L$. All our MC data are compatible with a behaviour of type (3).

The critical parameters can be computed with great accuracy studying the crossing of several quantities (second momentum correlation length divided by the lattice length, and the Binder parameter for the magnetization) for the different lattice sizes. The deviation of the crossing point for lattice sizes L, sL scales as

$$\Delta\beta_c^L, \Delta p_c^L \propto \frac{1-s^{-\omega}}{s^{1/\nu}-1} L^{-\omega-1/\nu}. \tag{4}$$

We refer to [10] for details of the method. The results are reported in table 3. Note that for the Ising model the method gives the correct value within errors for the critical temperature (1 part in 10^5). Regarding the ω exponent, whose conjectured value is $\omega = \frac{4}{3}$ [15] for the pure system, to obtain more accurate results, simulations on smaller lattices should be added, but this would increase the systematic errors in the determination of the critical coupling.

We finally measure the specific heat C_V at the critical points. Excluding the Ising limit, a linear function of $\log(\log L)$, is predicted in [2]. To control the effects of the uncertainty on the critical couplings in this kind of analysis, we have fitted our data at the critical coupling and also one standard deviation apart. We conclude that this source of error is negligible, given our accuracy in the critical points (χ^2/d.o.f. changes at most by 12%). The results are quite compatible with the theoretical prediction, as displayed in figure 2.

We thus conclude that the site-diluted Ising model in two dimensions belongs to the same universality class as the pure Ising model, although we find strong logarithmic effects.

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References

[1] Kim J K and Patrascioiu A 1994 *Phys. Rev. Lett.* **72** 2785
Kim J K and Patrascioiu A 1994 *Phys. Rev.* B **49** 15 764
[2] Dotsenko V S and Dotsenko Vi S 1983 *Adv. Phys.* **32** 129
[3] Andreichenko V B, Dotsenko Vi S, Selke W and Wang J S 1990 *Nucl. Phys.* B **344** 531
[4] Dotsenko V, Picco M and Pujol P 1995 Correlation functions for the 2D random bonds Potts models *Proc. Trieste Conf. on Recent Developments in Statistical Mechanics and Quantum Field Theory, Preprint* cond-mat/9509149
[5] Ballesteros H G, Fernández L A, Martín-Mayor V, Muñoz Sudupe A, Parisi G and Ruiz-Lorenzo J J 1997 *Preprint* hep-lat/9707017
[6] Aarão Reis F D A, de Queiroz S L A and dos Santos R R 1997 *Phys. Rev.* B **56** 6013
[7] Swendsen R H and Wang J S 1987 *Phys. Rev. Lett.* **58** 86
[8] Wolff U 1989 *Phys. Rev. Lett.* **62** 3834
[9] Stauffer D and Aharony A 1994 *Introduction to the Percolation Theory* (London: Taylor and Francis)
[10] Ballesteros H G, Fernández L A, Martín-Mayor V and Muñoz Sudupe A 1996 *Phys. Lett.* B **378** 207
Ballesteros H G, Fernández L A, Martín-Mayor V and Muñoz Sudupe A 1996 *Phys. Lett.* B **387** 125
Ballesteros H G, Fernández L A, Martín-Mayor V and Muñoz Sudupe A 1997 *Nucl. Phys.* B **483** 707
[11] Nienhuis B 1982 *J. Phys. A: Math. Gen.* **15** 199
[12] Ballesteros H G, Fernández L A, Martín-Mayor V, Muñoz Sudupe A, Parisi G and Ruiz-Lorenzo J J 1997 *Phys. Lett.* B **400** 346
[13] Ruiz-Lorenzo J J 1997 *J. Phys. A: Math. Gen.* **30** 485
[14] Shalaev B N 1984 *Sov. Phys. Solid State* **26** 1811
Shankar R 1987 *Phys. Rev. Lett.* **58** 2466
[15] Zinn-Justin J 1990 *Quantum Field Theory and Critical Phenomena* (Oxford: Clarendon)